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Thermalized displaced squeezed thermal states

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Abstract. In the coordinate representation of thermofield dynamics, we investigate the thermalized displaced squeezed thermal state which involves two temperatures successively. We give the wavefunction and the matrix element of the density operator at any time, and accordingly calculate some quantities related to the position and particle number operator, special cases of which are consistent with the results in the literature. The two temperatures have different correlations with the squeezing and coherence components. Moreover, different from the properties of the position, the average value and variance of the particle number operator as well as the second-order correlation function are time independent.

Introducing finite temperature effects into squeezed states (including squeezed vacuum, squeezed displaced and displaced squeezed states) is important, because a squeezed state can possesses minimum uncertainty, squeezability and accordingly technological applicability [1], and a finite-temperature influence on it is inevitable. This problem has received extensive investigations and a variety of squeezed states with finite temperature effects were constructed and investigated within different formalisms [2–11]. Based on them, [2, 3] classified these states into the thermalized squeezed states [4–6] and the squeezed thermal states [7–11]. Both of these two types of states are physically distinct states, although they can be transformed into each other by some parameter transformation [3]. Each of these two states has several possible representations, and [2] gave a detailed discussion about them and elucidated their physical interpretation. From [2], it is not difficult to understand that the squeezed thermal states correspond to the output from a squeezed device whose input is a thermal chaotic state with a Bose–Einstein distribution, while the thermalized squeezed states are prepared by thermalizing a squeezed state provided the thermalizing source is such that it can bring a vacuum state into a thermal chaotic state. Hence the two types of states represent two different ways of introducing a finite temperature into squeezed states: one is to introduce the thermal effects before squeezing and the other after squeezing, and each of them is close to practical cases (an absolutely pure squeezed state is impossible).

However, more practically and generally, perhaps we should consider thermal effects both before squeezing and after squeezing. From a theoretical viewpoint, since both the thermalized squeezed states and the squeezed thermal states have been investigated, it is natural and interesting for one to consider a general type of state which can take the above two types of states as its special cases. Furthermore, from a practical standpoint, such a general state is a more real state than the squeezed states in the literature. As a matter of fact, when a squeezing device acting on a ground state produces a squeezed state, not only the input but also the output would be mixed with thermal noise. For example, in the possible measurement of gravitational waves proposed by Caves [12], when a Michelson interferometer works with the light, the light will be the mixture of the laser light in a coherent state and the squeezed vacuum state from the output from a degenerate parameter amplifier whose input is a ground state [12]. Because of the inevitable existence of thermal noise, the input from the ground state will be a thermal chaotic state at some temperature, and also both the output from the degenerate parameter amplifier and the laser will be mixed with the environmental thermal noise whose temperature will perhaps be different from the temperature of the input. Thus, strictly speaking, the light reflected by the mirrors in the Michelson interferometer is the mixture of a coherent state with thermal noise and a squeezed state with thermal noise pre- and post-squeezing the ground state. For another example, in optical communication, consider light in a squeezed displaced state which will be the output from a squeezing device whose input is a coherent state [13]. It is evident that, in fact, the signal entering the squeezing device would be in a coherent state with thermal noise pre- and post-displacing the ground state, and in the course of the transmission the squeezed displaced thermal light would be mixed with thermal noise in the fibre. That is to say, the signal transmitted in the fibre should be in a squeezed displaced state with thermal noise pre-displacing and post-squeezing it. Yet another example is a trapped ion in a squeezed state. Recently, a trapped ion has been first cooled to a ground state and then stimulated to a coherent state as well as a squeezed vacuum state [14]. One can predict that in the not too distant future various squeezed states of a trapped ion will be prepared. Again owing to the thermal noise, a trapped ion in a squeezed state must be mixed with noise both before and after squeezing. In other words, for a squeezing device (including a combination of the squeezing and displacement devices), its input would be accompanied by thermal noise, its output would also encounter thermal noise, and hence to introduce thermal effects into a squeezed state both pre- and post-squeezing a ground state is necessary and more practical. Of course, perhaps the thermal effect will not be strong when the temperatures are very low, but no matter how weak it is in this case the thermal noise really exists before and after squeezing.

In this paper, we intend to construct such a general squeezed state with thermal effects which we call the thermalized displaced squeezed thermal state (TDSTS). Within the framework of thermofield dynamics [15, 16], we shall first give the definition of the thermalized displaced squeezed thermal state, and then give the time-dependent wavefunction and calculate the matrix elements of the density operator in the coordinate representation. From the density matrix elements, the probability density, the average value and variance of the position will be discussed. Finally, we shall also give the average value and variance of the particle number operator and the second-order correlation function.

This paper is based on a one-dimensional quantum oscillator with mass m and a constant angular frequency ω whose Hamiltonian is

$$H = \frac{1}{2m}\boldsymbol{p}^2 + \frac{1}{2}m\omega^2 x^2 = \left(a^\dagger a + \tfrac{1}{2}\right)\hbar\omega \tag{1}$$

where x is the position operator, $\boldsymbol{p} = -\mathrm{i}\hbar \mathrm{d}/\mathrm{d}x \equiv -\mathrm{i}\hbar\partial_x$ is the momentum operator in the coordinate representation, and

$$a = \frac{1}{\sqrt{2m\hbar\omega}}(\mathrm{i}\boldsymbol{p} + m\omega x) \qquad a^\dagger = \frac{1}{\sqrt{2m\hbar\omega}}(-\mathrm{i}\boldsymbol{p} + m\omega x) \tag{2}$$

are the annihilation and creation operators, respectively. Nevertheless, taking the mass as unity in the formulae of this paper, one can find results which are usable for a one-mode electromagnetic field with the same frequency.

As is well known, a displaced squeezed state can be constructed by the squeezing and displacement operators acting successively on the ground state $|0\rangle$ of the oscillator, equation (1) [17]. The displacement operator

$$D(\alpha) = \mathrm{e}^{\alpha a^{\dagger} - \alpha^{*} a} \tag{3}$$

with $\alpha = (\alpha_1 + \mathrm{i}\alpha_2) = |\alpha|\mathrm{e}^{\mathrm{i}\gamma}$ any complex number, corresponds to an ideal displacement device (the symbol '$|\cdots|$' represents the modulus of a complex number), and the squeezing operator

$$S(z) = \exp\left\{-\tfrac{1}{2}(z^{*}aa - za^{\dagger}a^{\dagger})\right\} \tag{4}$$

with $z = z_1 + \mathrm{i}z_2 = r\mathrm{e}^{\mathrm{i}\phi}$ any complex number, corresponds to an ideal squeezing device. (In equation (4), the minus '$-$' before '$\frac{1}{2}$' is sometimes replaced by a plus '+' in the literature, which gives rise to no essential differences.) The action of the displacement operator following the squeezing operator on the ground state will yield the displaced squeezed state $D(\alpha)S(z)|0\rangle$ (here the aforementioned squeezing device is really a combination of the displacement and squeezing devices), and a squeezed displaced state $S(z)D(\alpha)|0\rangle$ can also be constructed by the action of the squeezing operator following the displacement operator. Note that through a parameter transformation the states $D(\alpha)S(z)|0\rangle$ and $S(z)D(\alpha)|0\rangle$ can be transformed into each other, because one has [18]

$$S(z)D(\alpha) = D(\alpha\cosh(r) + \alpha^{*}\mathrm{e}^{\mathrm{i}\phi}\sinh(r))S(z). \tag{5}$$

So this paper involves the displaced squeezed states only.

In order to consider finite temperature effects, thermofield dynamics [15] introduces a copy of the physical oscillator equation (1) (called the tildian oscillator)

$$\tilde{H} = \frac{1}{2m}\tilde{\boldsymbol{p}}^2 + \frac{1}{2}m\omega^2\tilde{x}^2 = \left(\tilde{a}^{\dagger}\tilde{a} + \tfrac{1}{2}\right)\hbar\omega \tag{6}$$

according to the tildian 'conjugation': $\widetilde{C\boldsymbol{O}} \equiv C^{*}\tilde{\boldsymbol{O}}$ [15]. Here, C is any coefficient appeared in expressions of quantities for the physical system, $\boldsymbol{O}$ any operator, the superscript '$*$' denotes complex conjugation and $\tilde{\boldsymbol{O}}$ represents the corresponding operator for the tildian system. Based on the physical and tildian oscillators, thermofield dynamics manufactures a thermal operator $\mathcal{T}(\theta)$

$$\mathcal{T}(\theta) = \exp\{-\theta(\beta)(a\tilde{a} - a^{\dagger}\tilde{a}^{\dagger})\} = \exp\left\{\mathrm{i}\frac{\theta}{\hbar}(x\tilde{\boldsymbol{p}} - \tilde{x}\boldsymbol{p})\right\} \tag{7}$$

with

$$\tanh[\theta(\beta)] = \mathrm{e}^{-\beta\hbar\omega/2}$$

and $\beta = 1/k_b T$. Here, k_b is the Boltzmann constant and T is the temperature. The thermal operator is invariant under the tildian conjugation, i.e. $\tilde{\mathcal{T}}(\theta) = \mathcal{T}(\theta)$. Letting $\mathcal{T}(\theta)$ act on the direct product of the physical ground state $|0\rangle$ and the tildian ground state $|\tilde{0}\rangle$, one can manufacture a thermal vacuum. Notice that any physical operator commutes with any tildian operator, physical operators act on physical states only and similarly tildian operators on tildian states only. Consequently, thermal-vacuum average values in thermofield dynamics agree with canonical ensemble average values in statistical mechanics [15]. When the thermal operator acts on the direct product of the free electromagnetic field vacuum and its tildian counterpart, one can obtain a thermal chaotic state which describes a thermal chaotic light with a Bose–Einstein distribution at a finite temperature T. That is to say, generally, the thermal operator $\mathcal{T}(\theta)$ represents the action of a thermal source.

Now we can give the definition of the TDSTS. Since $\mathcal{T}(\theta)$ is involved in the physical and tildian operators simultaneously, we shall use both the physical operators $D(\alpha)$, $S(z)$ and their tildian counterparts $\tilde{D}(\alpha)$, $\tilde{S}(z)$ when a finite temperature effect is introduced into a displaced squeezed state (the tildian operators $\tilde{D}(\alpha)$ and $\tilde{S}(z)$ were not adopted in [2]). Because $\mathcal{T}(\theta)$ commutes with $S(z)\tilde{S}(z)$ [3] and does not commute with $D(\alpha)\tilde{D}(\alpha)$, we can define a state which involves two thermal sources with different temperatures successively. The following TDSTS,

$$|\beta_2, \alpha, z, \beta_1, 0\rangle \equiv \mathcal{T}(\theta_2)D(\alpha)\tilde{D}(\alpha)S(z)\tilde{S}(z)\mathcal{T}(\theta_1)|0\rangle|\tilde{0}\rangle \tag{8}$$

where β_1, θ_1 and β_2, θ_2 correspond to those at the temperatures T_1 and T_2, respectively. Furthermore, the time evolution of the TDSTS can be considered by the evolution operator $\hat{U}(t) = \exp\{-(\mathrm{i}/\hbar)(H - \tilde{H})\}$ acting on the TDSTS, i.e.

$$|t, \beta_2, \alpha, z, \beta_1, 0\rangle = \hat{U}(t)|\beta_2, \alpha, z, \beta_1, 0\rangle \tag{9}$$

where t is the time. Obviously, the displaced squeezed thermal state (DSTS) [2, 3, 9] is the TDSTS for $T_2 = 0$, the thermalized displaced squeezed state (TDSS) for $T_1 = 0$, and the thermalized coherent thermal state for $z = 0$. The TDSTS equation (8) with proper parameter constraints and transformation can be reduced to almost all the cases discussed in [2–11] except for the displaced thermalized squeezed states defined by a density matrix in [2]. The TDSTS can be prepared by thermalizing a DSTS [9] (second reference), and can describe the practical examples listed in the second paragraph. The TDSTS involves two thermal sources successively and contains both the pre- and post-displacing squeezing thermal noise at T_1 and T_2, respectively. For the sake of convenience, we call the thermal noise at T_2 the detector thermal noise and the thermal noise at T_1 the input thermal noise.

In the coordinate representation, all the displacement, squeezing and thermal operators can be untangled [19, 20]. Moreover, if we exploit the thermal coordinate representation introduced in [20], one can easily untangle the evolution operator $\hat{U}(t)$. Hence one can obtain the explicit expression of the time-dependent wavefunction $\langle\tilde{x}, x|t, \beta_2, \alpha, z, \beta_1, 0\rangle$. Alternatively, noticing that $D(\alpha)\tilde{D}(\alpha)\mathcal{T}(\theta) = \mathcal{T}(\theta)D(\alpha(\cosh(\theta) - \sinh(\theta)))\tilde{D}(\alpha(\cosh(\theta) - \sinh(\theta)))$ [21], one has $|t, \beta_2, \alpha, z, \beta_1, 0\rangle = |t, \beta_2, \beta_1, \alpha(\cosh(\theta_1) - \sinh(\theta_1)), z, 0\rangle$ and can easily obtain the time-dependent wavefunction with the help of the result in [20]. Taking $n = 0$, $\theta = \theta_1 + \theta_2$ and the replacement $\alpha \to \alpha(\cosh(\theta_1) - \sinh(\theta_1))$ in equation (44) of [20], one can read

$$\begin{aligned}
\langle\tilde{x}, x|t, \beta_2, \alpha, z, \beta_1, 0\rangle = {} & \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \frac{1}{|\mathcal{F}_1 B|} \exp\left\{-\frac{Q}{B} - \frac{Q^*}{B^*}\right\} \\
& \times \exp\left\{-\frac{m\omega}{2\hbar}G_1(x\cosh(\Theta) - \tilde{x}\sinh(\Theta))^2\right. \\
& \left. +2\sqrt{\frac{m\omega}{2\hbar}}G_2(\cosh(\theta_1) - \sinh(\theta_1))(x\cosh(\Theta) - \tilde{x}\sinh(\Theta))\right\} \\
& \times \exp\left\{-\frac{m\omega}{2\hbar}G_1^*(\tilde{x}\cosh(\Theta) - x\sinh(\Theta))^2\right. \\
& \left. +2\sqrt{\frac{m\omega}{2\hbar}}G_2^*(\cosh(\theta_1) - \sinh(\theta_1))(\tilde{x}\cosh(\Theta) - x\sinh(\Theta))\right\}
\end{aligned} \tag{10}$$

where

$$\mathcal{F}_1 = \cosh(r) + \sinh(r)\cos(\phi) + \mathrm{i}\sinh(r)\sin(\phi)$$
$$\mathcal{F}_2 = \frac{1 - \mathrm{i}\sinh(2r)\sin(\phi)}{\cosh(2r) + \sinh(2r)\cos(\phi)}$$
$$\Theta = \theta_1 + \theta_2 \qquad B = \cos(\omega t) + \mathrm{i}\mathcal{F}_2\sin(\omega t)$$
$$G_1 = \frac{\mathcal{F}_2\cos(\omega t) + \mathrm{i}\sin(\omega t)}{B} \qquad G_2 = \frac{\mathcal{F}_2\alpha_1 + \mathrm{i}\alpha_2}{B}$$

and

$$Q = [\mathcal{F}_2\cos(\omega t)\,\alpha_1^2 + 2\mathcal{F}_2\sin(\omega t)\,\alpha_1\alpha_2 + \mathrm{i}\sin(\omega t)\,\alpha_2^2](\cosh(\theta_1) - \sinh(\theta_1))^2.$$

Equation (10) is the time-dependent wavefunction of the TDSTS in the coordinate representation, and all information of the TDSTS can be extracted from it.

With the help of equation (10), a straightforward calculation yields the density matrix element $\rho_{x',x}(t)$ on the position for the TDSTS

$$\begin{aligned}
\rho_{x',x}(t) &\equiv \int_{-\infty}^{\infty} \langle\tilde{x}, x|t, \beta_2, \alpha, z, \beta_1, 0\rangle\langle 0, \beta_1, z, \alpha, \beta_2, t|x', \tilde{x}\rangle\,\mathrm{d}\tilde{x} \\
&= \sqrt{\frac{m\omega}{\pi\hbar}}\frac{1}{|\mathcal{F}_1 B|}\sqrt{\frac{1}{\cosh(2\Theta)}}\exp\left\{\frac{|\mathcal{F}_1 B|^2}{2\cosh(2\Theta)}\coth\left(\tfrac{1}{4}\beta_2\hbar\omega\right)(G_2 - G_2^*)^2\right\} \\
&\quad\times\exp\left\{-\frac{m\omega}{4\hbar}\frac{1}{|\mathcal{F}_1 B|^2\cosh(2\Theta)}\left[x + x' - \sqrt{\frac{2\hbar}{m\omega}}\sqrt{\coth\left(\tfrac{1}{4}\beta_2\hbar\omega\right)}\left(\frac{\alpha}{A} + \frac{\alpha^*}{A^*}\right)\right]^2\right. \\
&\quad - \frac{m\omega}{4\hbar}\frac{\cosh(2\Theta)}{|\mathcal{F}_1 B|^2}\left[x - x' - \sqrt{\frac{2\hbar}{m\omega}}\frac{|\mathcal{F}_1 B|^2}{\cosh(2\Theta)}\sqrt{\coth\left(\tfrac{1}{4}\beta_2\hbar\omega\right)}(G_2 - G_2^*)\right]^2 \\
&\quad\left. - \frac{m\omega}{4\hbar}(G_1 - G_1^*)(x^2 - x'^2)\right\}
\end{aligned} \tag{11}$$

with $A = \cos(\omega t) + \mathrm{i}\sin(\omega t)$. This density matrix is complex, Hermitian and time dependent. When $z = 0$ equation (11) gives the density matrix of the position for thermalized coherent thermal states $\mathcal{T}(\theta_2)D(\alpha)\tilde{D}(\alpha)\mathcal{T}(\theta)|0\rangle|\tilde{0}\rangle$ which contains the thermalized coherent state and the coherent thermal state as its special cases. At the initial time $t = 0$ and $T_2 = 0$, equation (11) is reduced to the density matrix element of the position for the DSTS,

$$\begin{aligned}
\rho_{x',x} &= \sqrt{\frac{m\omega}{\pi\hbar}}\frac{1}{|\mathcal{F}_1|}\sqrt{\frac{1}{\cosh(2\theta_1)}}\exp\left\{\frac{|\mathcal{F}_1|^2}{2\cosh(2\theta_1)}[(\mathcal{F}_2 - \mathcal{F}_2^*)\alpha_1 + \mathrm{i}2\alpha_2]^2\right\} \\
&\quad\times\exp\left\{-\frac{m\omega}{4\hbar}\frac{1}{|\mathcal{F}_1|^2\cosh(2\theta_1)}\left[x + x' - \sqrt{\frac{2\hbar}{m\omega}}2\alpha_1\right]^2\right. \\
&\quad - \frac{m\omega}{4\hbar}(\mathcal{F}_2 - \mathcal{F}_2^*)(x^2 - x'^2) \\
&\quad\left. - \frac{m\omega}{4\hbar}\frac{\cosh(2\theta_1)}{|\mathcal{F}_1|^2}\left[x - x' - \sqrt{\frac{2\hbar}{m\omega}}\frac{|\mathcal{F}_1|^2}{\cosh(2\theta_1)}((\mathcal{F}_2 - \mathcal{F}_2^*)\alpha_1 + \mathrm{i}2\alpha_2)\right]^2\right\}
\end{aligned} \tag{12}$$

which is identical to equation (6.5a) in [9] (second reference). Of course, setting $T_1 = 0$, one can obtain the density matrix element of the TDSS. Noticing

$$\cosh(2\Theta) = \coth\left(\tfrac{1}{2}\beta_1\hbar\omega\right)\coth\left(\tfrac{1}{2}\beta_2\hbar\omega\right) + \mathrm{cosech}\left(\tfrac{1}{2}\beta_1\hbar\omega\right)\mathrm{cosech}\left(\tfrac{1}{2}\beta_1\hbar\omega\right) \tag{13}$$

and $\cosh(\theta) = \coth\left(\tfrac{1}{2}\beta\hbar\omega\right)$, one can see that the differences both among the density matrices of the TDSTS, the DSTS as well as the TDSS and among the finite temperature influences on these states consist in the appearance or disappearance of the four factors $\cosh(2\Theta), \cosh(2\theta_1), \cosh(2\theta_2)$ and $\coth\left(\frac{1}{4}\beta_2\hbar\omega\right)$. In the expression of the density matrices, the factors $\cosh(2\Theta), \cosh(2\theta_1)$ and $\coth\left(\frac{1}{4}\beta_2\hbar\omega\right)$ appear for the TDSTS, the factors $\cosh(2\theta_2)$ and $\coth\left(\frac{1}{4}\beta_2\hbar\omega\right)$ for the TDSS, and for the DSTS the factor $\cosh(2\theta_1)$ replaces $\cosh(2\Theta)$.

Taking $x' = x$ in equation (11), we have the probability density of the position for the TDSTS

$$\rho_{x,x}(t) = \sqrt{\frac{m\omega}{\pi\hbar}}\frac{1}{|\mathcal{F}_1 B|}\sqrt{\frac{1}{\cosh(2\Theta)}}\exp\left\{-\frac{m\omega}{\hbar}\frac{1}{|\mathcal{F}_1 B|^2\cosh(2\Theta)}\right. \\ \left.\times\left[x - \sqrt{\frac{\hbar}{2m\omega}}\sqrt{\coth\left(\tfrac{1}{4}\beta_2\hbar\omega\right)}\left(\frac{\alpha}{A}+\frac{\alpha^*}{A^*}\right)\right]^2\right\}. \tag{14}$$

This is a Gaussian distribution. From the probability density one can easily find the average value of the position

$$\langle x\rangle \equiv \int_{-\infty}^{\infty} x\rho_{x,x}\,\mathrm{d}x = \sqrt{\frac{2\hbar}{m\omega}}\sqrt{\coth\left(\tfrac{1}{4}\beta_2\hbar\omega\right)}\,|\alpha|\cos(\omega t-\gamma) \tag{15}$$

and the position variance

$$(\Delta x)^2 \equiv \langle x^2\rangle - \langle x\rangle^2 = \frac{\hbar}{2m\omega}[\cosh(2r)+\sinh(2r)\cos(2\omega t-\phi)]\cosh(2\Theta). \tag{16}$$

Here and subsequently, '$\langle\cdots\rangle$' denotes '$\langle 0, \beta_1, z, \alpha, \beta_2, t|\cdots|t, \beta_2, \alpha, z, \beta_1, 0\rangle$'.

From the above results of the position, one can easily write off the corresponding formulae of the momentum by comparing the momentum representation with the coordinate representation. Here we do not intend to consider them further.

The squeezing effect can be discussed using the rotated quadrature phase operators (the definitions here are slightly different from those in both [2] and [17] (the book))

$$Y_1 = \frac{1}{2}(a\mathrm{e}^{-\mathrm{i}\varphi} + a^\dagger\mathrm{e}^{\mathrm{i}\varphi}) \qquad Y_2 = \frac{1}{2\mathrm{i}}(a\mathrm{e}^{-\mathrm{i}\varphi} - a^\dagger\mathrm{e}^{\mathrm{i}\varphi}) \tag{17}$$

which give the quadrature phase operators $X_1 = \sqrt{m\omega/2\hbar}\,x$ and $X_2 = \sqrt{1/2m\hbar\omega}\,\boldsymbol{p}$ when the rotated angle $\varphi = 0$. A straightforward calculation yields

$$(\Delta Y_1)^2 \equiv \langle Y_1^2\rangle - \langle Y_1\rangle^2 = \tfrac{1}{4}\cosh(2\Theta)[\cosh(2r)+\sinh(2r)\cos(2\omega t+2\varphi-\phi)] \tag{18}$$

and

$$(\Delta Y_2)^2 \equiv \langle Y_2^2\rangle - \langle Y_2\rangle^2 = \tfrac{1}{4}\cosh(2\Theta)[\cosh(2r)-\sinh(2r)\cos(2\omega t+2\varphi-\phi)]. \tag{19}$$

The last two equations indicate that for the TDSTS, it is always possible to attenuate either ΔY_1 or ΔY_2 at any value of ϕ and any time t. At $t = 0$, equations (18) and (19) are identical to equation (6.9) in [9] (second reference) when $\varphi = 0$, $T_2 = 0$, and to equation (3.7) in [2] when $T_2 = 0$ and $(\phi - 2\varphi) = \pi$.

We have discussed the properties related to the position and quadrature phase. Next, we will give the average value and variance of the particle number operator $\boldsymbol{n} = \left\{(\hbar/2m\omega)\left[(m\omega/\hbar)^2x^2 - \partial_x^2\right] - \frac{1}{2}\right\}$ and the second-order correlation function. The average value of $\boldsymbol{n}$ at any time t is

$$\langle\boldsymbol{n}\rangle = \tfrac{1}{2}\cosh(2\Theta)\cosh(2r) - \tfrac{1}{2} + \coth\left(\tfrac{1}{4}\beta_2\hbar\omega\right)|\alpha|^2. \tag{20}$$

Thus, the input thermal noise is correlated only with the squeezing, while the detector thermal noise takes effects from both the squeezing and the coherence. When $T_2 = 0$, equation (20) is equation (3.9) in [9](1993) and equation (3.3) in [2]. Exploiting equations (18) and (20), one can calculate the variance of $\boldsymbol{n}$ at any time and obtain

$$(\Delta n)^2 \equiv \langle \boldsymbol{n}^2 \rangle - \langle \boldsymbol{n} \rangle^2 = -\tfrac{1}{4} + \tfrac{1}{4}\cosh^2(2\Theta)\cosh(4r) + \cosh(2\Theta)\coth\left(\tfrac{1}{4}\beta_2\hbar\omega\right)|\alpha|^2[\cosh(2r) + \sinh(2r)\cos(\phi - 2\gamma)]. \quad (21)$$

Consequently, one has the second-order correlation function at any time t,

$$g^{(2)}(0) = \frac{\langle \boldsymbol{n}^2 \rangle - \langle \boldsymbol{n} \rangle}{\langle \boldsymbol{n} \rangle^2} = 2 + \left[\tfrac{1}{4}\cosh^2(2\Theta)\sinh^2(2r) - \coth^2\left(\tfrac{1}{4}\beta_2\hbar\omega\right)|\alpha|^4 + \cosh(2\Theta)\coth\left(\tfrac{1}{4}\beta_2\hbar\omega\right)|\alpha|^2\sinh(2r)\cos(\phi - 2\gamma)\right] \times\left[\tfrac{1}{2}\cosh(2\Theta)\cosh(2r) - \tfrac{1}{2} + \coth\left(\tfrac{1}{4}\beta_2\hbar\omega\right)|\alpha|^2\right]^{-2}. \quad (22)$$

When $T_2 = 0$ the last equation is identical to equation (3.10) in [9] (second reference). From equations (20)–(22), one sees that the average value and variance of the particle number operator and the second-order correlation function are time independent.

In conclusion, this paper has investigated the TDSTS which is a generalization of the TDSS and the DSTS. In the coordinate representation of thermofield dynamics, we give the wavefunction and the density matrix with time evolution, and calculate some quantities related to the position, quadrature phase and particle number at any time. Our results indicates that the quantities related to the position and the momentum are time dependent, but the average value and variance of the particle number operator and the second-order correlation function are time independent. For the influence of the temperature on the displaced squeezed state, the TDSTS possesses the features both of the TDSS and of the DSTS, but the finite-temperature effects of the TDSTS are not a simple sum or product of those of the TDSS and the DSTS. The input thermal noise at T_1 influences only the squeezing component via the factor $\cosh(2\Theta)$, while the detector thermal noise at T_2 has an additional effect on the coherence component by the factor $\coth\left(\frac{1}{4}\beta_2\hbar\omega\right)$. The TDSTS takes various thermal squeezed states in the literature as its special cases, except for the density-matrix-defined displaced thermalized squeezed state in [2]. For the definition equation (8), if inserting the thermal operators $\mathcal{T}(\theta_3)$, $\mathcal{T}(\theta_4)$ and $\mathcal{T}(\theta_5)$ between $D(\alpha)$, $\tilde{D}(\alpha)$, $S(z)$ and $\tilde{S}(z)$, respectively, without self-tildian requirement (as [4] (first reference) did), one can obtain a most generalized displaced squeezed state with finite-temperature effects. However, because such a state is not invariant under the interchange between $D(\alpha)$ and $\tilde{D}(\alpha)$ or $S(z)$ and $\tilde{S}(z)$, we believe it is perhaps not meaningful. Thus, we should say, within the framework of thermofield dynamics, that the TDSTS of this paper is the most generalized squeezed state with thermal effects. Finally, we want to point out that, if there are M input thermal noise and N detector thermal noise and the corresponding temperatures are $T_{1,1}, T_{1,2}, \ldots, T_{1,M}$ and $T_{2,1}, T_{2,2}, \ldots, T_{2,N}$, then the physical quantities of such a generalized TDSTS can be given by taking $\Theta = \theta_{1,1} + \theta_{1,2} + \cdots + \theta_{1,M} + \theta_{2,1} + \theta_{2,2} + \cdots + \theta_{2,N}$ and $\theta_2 = \theta_{2,1} + \theta_{2,2} + \cdots + \theta_{2,N}$ in the results of this paper. We believe that once squeezed states are successfully being used in optical communication and sensitive measurements, the discussions of the present paper will be found to be useful.

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